

Exercise	1	2	3	4	Total
100%	3	3	3	9	18
Points					

## Extragalactic Astronomy and Cosmology

Homework 11 - Lecture 25 - The Universe

**Due date: December 12**

To contact me: e-mail to [beckmann@milkyway.gsfc.nasa.gov](mailto:beckmann@milkyway.gsfc.nasa.gov) or visit me during office hours (Tuesdays & Thursdays 10 - 12:30 a.m.) in room physics 415

This final homework assignment is part of the preparation for the final exam on Thursday, December 14, from 8 a.m. to 10 a.m. in room 201 (our usual class room). Do not forget to bring your calculator, pen, and paper (although I'll try to get Blue Books). It is foreseen that you can pick up the corrected exam together with a note about your final grade (both in a closed envelope of course) for PHYS 316 in the physics office room 220 already on Friday, December 15.

### 1 Friedmann equation

This is the Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_{\omega} \epsilon_{\omega,0} a^{-1-3\omega} - \frac{\kappa c^2}{R_0^2} \quad (1)$$

What is  $\omega$  ? What possible values can  $\omega$  have ?

What is  $\kappa$  ? How is it related to the possible geometries of the Universe ?  
What is  $\kappa$  as a function of time ?

### 2 Receding galaxies

Can galaxies recede away from us faster than light? If yes, why does this not break the speed limit set by the speed of light? At what distance is a galaxy receding exactly with the speed of light?

### 3 Acceleration equation

This is the acceleration equation:  $\ddot{a} = -\frac{4\pi G}{3c^2} (\epsilon + 3P) + \frac{\Lambda}{3}$

What is  $\ddot{a}$  ? If  $\epsilon > 0$  and  $P > 0$ , how is the expansion rate changing with time? If in a given setting,  $P$  is increased, does the expansion accelerate or decelerate?

## 4 Can neutrinos close the Universe?

The cosmic microwave background at 2.725 Kelvin contributes an energy density corresponding to a density parameter of  $\Omega_{CMB,0} = 2.5 \times 10^{-5}$ . The typical energy of a photon of light in a thermal distribution is given by  $3k_B T$  (where  $k_B = 8.6 \times 10^{-5} \text{ eV K}^{-1}$ ). Suppose that the number of neutrinos matches the number of photons. What mass-energy (in electron Volts) would these neutrinos have to have in order to contribute a critical density? (Assume that the thermal energy of the neutrinos is negligible to their mass-energy). The present upper limit of the electron neutrino mass-energy from experiments on Earth is about 10 eV. How low would  $H_0$  have to be to enable electron neutrinos to contribute all the dark matter in a Universe with the critical density?

A more accurate calculation suggests that the neutrino mass-energy required to give the critical density is larger than the crude calculation above suggests, being about 90 eV. Is the electron neutrino a realistic dark matter candidate?